Elementary maths

This is a collection of basic mathematical computations using `sympy`. The main purpose is to demonstrate the use of `\py` and `\py*`. Note that `sympy 1.1.1` appears unable to simplify $\tanh(\log(x))$ (compare `rhs.108` shown below against `ans.108` shown in the Mathematica examples). Note also the separate computaions for the left and right hand sides of results 108, 109 and 110.

```
from sympy import *
x, y, z, a, b, c = symbols('x y z a b c')
ans = expand((a+b)**3)                                # py (ans.101,ans)
ans = factor(-2*x+2*x+a*x-x**2+a*x**2-x**3)           # py (ans.102,ans)
ans = solve(x**2-4, x)                                # py (ans.103,ans)
ans = solve([2*a-b - 3, a+b+c - 1,-b+c - 6],[a,b,c])  # py (ans.104,ans)
ans = N(pi,50)                                        # py (ans.105,ans)
ans = apart(1/((1 + x)*(5 + x)))                      # py (ans.106,ans)
ans = together((1/(1 + x) - 1/(5 + x))/4)             # py (ans.107,ans)
ans = simplify(tanh(log(x)))                          # py (rhs.108,ans)
ans = simplify(tanh(I*x))                             # py (rhs.109,ans)
ans = simplify(sinh(3*x) - 3*sinh(x) - 4*(sinh(x))**3) # py (rhs.110,ans)
ans = tanh(log(x))                                    # py (lhs.108,ans)
ans = tanh(UnevaluatedExpr(I*x))                      # py (lhs.109,ans)
ans = sinh(3*x) - 3*sinh(x) - 4*(sinh(x))**3          # py (lhs.110,ans)
```

```
\begin{align*}
   &\py*{ans.101}\\
   &\py*{ans.102}\\
   &\py*{ans.103}\\
   &\py*{ans.104}\\
   &\py*{ans.105}\\
   &\py*{ans.106}\\
   &\py*{ans.107}\\
   \py{lhs.108} &= \Py{rhs.108}\\
   \py{lhs.109} &= \Py{rhs.109}\\
   \py{lhs.110} &= \Py{rhs.110}
\end{align*}
```

$$\texttt{ans.101} := a^3 + 3a^2b + 3ab^2 + b^3$$

$$\texttt{ans.102} := -x\left(-a+x\right)\left(x+1\right)$$

$$\texttt{ans.103} := \left[-2, \quad 2\right]$$

$$\texttt{ans.104} := \left\{a : \frac{1}{5}, \quad b : -\frac{13}{5}, \quad c : \frac{17}{5}\right\}$$

$$\texttt{ans.105} := 3.1415926535897932384626433832795028841971693993751$$

$$\texttt{ans.106} := -\frac{1}{4\left(x+5\right)} + \frac{1}{4\left(x+1\right)}$$

$$\texttt{ans.107} := \frac{1}{\left(x+1\right)\left(x+5\right)}$$

$$\tanh\left(\log\left(x\right)\right) = \tanh\left(\log\left(x\right)\right) \tag{rhs.108}$$

$$\tanh\left(ix\right) = i\tan\left(x\right) \tag{rhs.109}$$

$$-4\sinh^3\left(x\right) - 3\sinh\left(x\right) + \sinh\left(3x\right) = 0 \tag{rhs.110}$$

Linear Algebra

```
from sympy import linsolve
lamda = Symbol('lamda')
mat  = Matrix([[2,3], [5,4]])                # py (ans.201,mat)
eig1 = mat.eigenvects()[0][0]                # 1st eigenvalue
eig2 = mat.eigenvects()[1][0]                # 2nd eigenvalue
v1   = mat.eigenvects()[0][2][0]             # 1st eigenvector
v2   = mat.eigenvects()[1][2][0]             # 2nd eigenvector
eig  = simplify(Matrix([eig1,eig2]))         # py (ans.202,eig)
vec  = simplify(5*Matrix([]).col_insert(0,v1)
                          .col_insert(1,v2)) # py (ans.203,vec)
det  = expand((mat - lamda * eye(2)).det())  # py (ans.204,det)
rhs  = Matrix([[3],[7]])                     # py (ans.205,rhs)
ans  = list(linsolve((mat,rhs),x,y))[0]      # py (ans.206,ans)
```

```
\begin{align*}
   &\py*{ans.201}\\
   &\py*{ans.202}\\
   &\py*{ans.203}\\
   &\py*{ans.204}\\
   &\py*{ans.205}\\
   &\py*{ans.206}
\end{align*}
```

$$\begin{aligned}
&\texttt{ans.201} := \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}\\
&\texttt{ans.202} := \begin{bmatrix} -1 \\ 7 \end{bmatrix}\\
&\texttt{ans.203} := \begin{bmatrix} -5 & 3 \\ 5 & 5 \end{bmatrix}\\
&\texttt{ans.204} := \lambda^2 - 6\lambda - 7\\
&\texttt{ans.205} := \begin{bmatrix} 3 \\ 7 \end{bmatrix}\\
&\texttt{ans.206} := \left(\frac{9}{7}, \quad \frac{1}{7}\right)
\end{aligned}$$

Limits

```
n, dx = symbols('n dx')
ans = limit(sin(4*x)/x,x,0)                    # py (ans.301,ans)
ans = limit(2**x/x,x,oo)                       # py (ans.302,ans)
ans = limit(((x+dx)**2 - x**2)/dx, dx,0)       # py (ans.303,ans)
ans = limit((4*n + 1)/(3*n - 1),n,oo)          # py (ans.304,ans)
ans = limit((1+(a/n))**n,n,oo)                 # py (ans.305,ans)
```

```
\begin{align*}
   &\py*{ans.301}\\
   &\py*{ans.302}\\
   &\py*{ans.303}\\
   &\py*{ans.304}\\
   &\py*{ans.305}
\end{align*}
```

$$\begin{aligned}
&\texttt{ans.301} := 4\\
&\texttt{ans.302} := \infty\\
&\texttt{ans.303} := 2x\\
&\texttt{ans.304} := \frac{4}{3}\\
&\texttt{ans.305} := e^{a}
\end{aligned}$$

Series

```
ans = series((1 + x)**(-2), x, 1, 6)           # py (ans.401,ans)
ans = series(exp(x), x, 0, 6)                  # py (ans.402,ans)
ans = Sum(1/n**2, (n,1,50)).doit()             # py (ans.403,ans)
ans = Sum(1/n**4, (n,1,oo)).doit()             # py (ans.404,ans)
```

```
\begin{align*}
   &\py*{ans.401}\\
   &\py*{ans.402}\\
   &\py*{ans.403}\\
   &\py*{ans.404}
\end{align*}
```

$$\begin{aligned}
&\texttt{ans.401} := \frac{1}{2} + \frac{3\left(x-1\right)^{2}}{16} - \frac{\left(x-1\right)^{3}}{8} + \frac{5\left(x-1\right)^{4}}{64} - \frac{3\left(x-1\right)^{5}}{64} - \frac{x}{4} + O\left(\left(x-1\right)^{6}; x \rightarrow 1\right)\\
&\texttt{ans.402} := 1 + x + \frac{x^{2}}{2} + \frac{x^{3}}{6} + \frac{x^{4}}{24} + \frac{x^{5}}{120} + O\left(x^{6}\right)\\
&\texttt{ans.403} := \frac{3121579929551692678469635660835626209661709}{1920815367859463099600511526151929560192000}\\
&\texttt{ans.404} := \frac{\pi^{4}}{90}
\end{aligned}$$

Calculus

This example shows how `\Py` can be used to set the equation tag on the far right hand side.

```
ans = diff(x*sin(x),x)                                  # py (ans.501,ans)
ans = diff(x*sin(x),x).subs(x,pi/4)                     # py (ans.502,ans)
ans = integrate(2*sin(x)**2, (x,a,b))                   # py (ans.503,ans)
ans = Integral(2*exp(-x**2), (x,0,oo))                  # py (lhs.504,ans)
ans = ans.doit()                                        # py (ans.504,ans)
ans = Integral(Integral(x**2 + y**2, (y,0,x)), (x,0,1)) # py (lhs.505,ans)
ans = ans.doit()                                        # py (ans.505,ans)
```

```
\begin{align*}
   &\py*{ans.501}\\
   &\py*{ans.502}\\
   &\py*{ans.503}\\
    \py{lhs.504}&=\Py{ans.504}\\
    \py{lhs.505}&=\Py{ans.505}
\end{align*}
```

$$\texttt{ans.501} := x\cos(x) + \sin(x)$$

$$\texttt{ans.502} := \frac{\sqrt{2}\pi}{8} + \frac{\sqrt{2}}{2}$$

$$\texttt{ans.503} := -a + b + \sin(a)\cos(a) - \sin(b)\cos(b)$$

$$\int_0^\infty 2e^{-x^2}\,dx = \sqrt{\pi} \tag{ans.504}$$

$$\int_0^1 \int_0^x \left(x^2 + y^2\right)\,dy\,dx = \frac{1}{3} \tag{ans.505}$$

Differential equations

```
y = Function('y')
C1, C2 = symbols('C1 C2')

ode = Eq(y(x).diff(x) + y(x), 2*a*sin(x))
sol = expand(dsolve(ode,y(x)).rhs)                              # py (ans.601,sol)
cst = solve([sol.subs(x,0)],dict=True)
sol = sol.subs(cst[0])                                          # py (ans.602,sol)

ode = Eq(y(x).diff(x,2) + y(x), 0)
sol = expand(dsolve(ode,y(x)).rhs)                              # py (ans.603,sol)
cst = solve([sol.subs(x,0),sol.diff(x).subs(x,0)-1],dict=True)
sol = sol.subs(cst[0])                                          # py (ans.604,sol)

ode = Eq(y(x).diff(x,2) + 5*y(x).diff(x) - 6*y(x), 0)
sol = expand(dsolve(ode,y(x)).rhs)                              # py (ans.605,sol)
sol = sol.subs({C1:2,C2:3})                                     # py (ans.606,sol)
```

```
\begin{align*}
   &\py*{ans.601}\\
   &\py*{ans.602}\\
   &\py*{ans.603}\\
   &\py*{ans.604}\\
   &\py*{ans.605}\\
   &\py*{ans.606}
\end{align*}
```

$$\begin{aligned}
&\texttt{ans.601} := C_1 e^{-x} + a\sin(x) - a\cos(x)\\
&\texttt{ans.602} := a\sin(x) - a\cos(x) + ae^{-x}\\
&\texttt{ans.603} := C_1\sin(x) + C_2\cos(x)\\
&\texttt{ans.604} := \sin(x)\\
&\texttt{ans.605} := C_1 e^{-6x} + C_2 e^{x}\\
&\texttt{ans.606} := 3e^{x} + 2e^{-6x}
\end{aligned}$$